Curved Diffusing Annulus Turbulent Boundary-Layer Development

SHOICHI FUJII*
National Aerospace Laboratory, Chofu, Tokyo, Japan
AND
THEODORE H. OKIISHI†
Iowa State University, Ames, Iowa

Nomenclature

c_f = local coefficient of friction, $\tau_0/\frac{1}{2}\rho U_{p,\delta}{}^2$,
$\bar{H}$ = energy thickness shape factor for flat-plate flow, δ^{**}/θ
H = shape factor for flat-plate flow, δ^*/θ
H_1 = shape factor for axisymmetric flow, $\delta_1{}^*/\theta_1$
r = radial coordinate
r_o = inner or outer casing radius
r_l = casing longitudinal radius of curvature
R_θ = Reynolds number based on momentum thickness, $U\theta/\nu$
U = freestream velocity for flat-plate flow
$U_{p,\delta}$ = mean velocity at the potential flow region, boundary-layer interface
u = mean local velocity within the boundary layer
x = coordinate along the casing or flat plate
y = coordinate normal to the casing or flat plate
z = axial coordinate
β = angle between y and r
δ = flat-plate flow boundary-layer thickness
δ_y = boundary-layer thickness along a normal line
δ_r = boundary-layer thickness along a radius
δ^{**} = flat-plate flow dissipation energy thickness,

$$\int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right)dy$$

δ^* = flat-plate flow displacement thickness, $\int_0^\delta \left(1-\frac{u}{U}\right)dy$

$\delta_1{}^*$ = axisymmetric flow displacement thickness,

$$\int_0^{\delta_y} \frac{r}{r_0}\left(1-\frac{u}{U_{p,\delta}}\right)dy$$

θ = flat-plate flow momentum thickness, $\int_0^\delta \left(\frac{u}{U}\right)\left(1-\frac{u}{U}\right)dy$

θ_1 = axisymmetric flow momentum thickness,

$$\int_0^{\delta_y} \frac{r}{r_0}\left(1-\frac{u}{U_{p,\delta}}\right)\frac{u}{U_{p,\delta}}\,dy$$

ν = kinematic viscosity
ρ = fluid density
τ_0 = shear stress at the casing

Received March 12, 1971; synoptic received September 16, 1971; revision received October 18, 1971. Full paper (Authors' Rept. No. ISU-ERI-Ames-71033) available from National Technical Information Service, Springfield, Va., 22151, as N72-10239 at the standard price (available upon request). This work was supported by the Engineering Research Institute at Iowa State University and the National Aerospace Laboratory, Tokyo, Japan.

Index category: Boundary Layers and Convective Heat Transfer-Turbulent.

* Chief Staff, Aero-Engines Div.

† Assistant Professor, Department of Mechanical Engineering and Engineering Research Institute.

Theme

THE turbulent boundary-layer development along the walls of an internal flow passage is an important facet of turbomachinery design and analysis. No entirely satisfactory general method of calculating the end wall boundary-layer development associated with the complicated flow through a turbomachine is currently available in the open literature. From an engineering point of view, it seems as if simplified but reliable methods of boundary-layer calculation can be obtained with acceptable assumptions for special flow situations; for example, several attempts in this direction have already been made[1-3] for axial flow compressors. The authors are not aware of similar work related to flow through curved-wall annular diffusing passages. In this paper, the von Kármán momentum integral approach for boundary-layer development calculation is used for predicting curved-wall annular diffuser boundary-layer growth.

Contents

A flow model consisting of developing annulus wall boundary layers which surround a potential flow core was adopted for the present calculations. The appropriate form of the von Kármán momentum integral equation to use is[4]

$$\frac{1}{r_o}\frac{d}{dx}(r_o\theta_1)+\frac{2+H_1}{U_{p,\delta}}\frac{dU_{p,\delta}}{dx}\theta_1=\frac{c_f}{2}+\frac{\nu\sin^2\beta}{r_oU^2}\int_0^{\delta_y}\frac{u}{r}\,dy \quad (1)$$

where

$$\theta_1=\int_0^{\delta_y}\frac{r}{r_o}\left(1-\frac{u}{U_{p,\delta}}\right)\frac{u}{U_{p,\delta}}\,dy,\quad \delta_1{}^*=\int_0^{\delta_y}\frac{r}{r_o}\left(1-\frac{u}{U_{p,\delta}}\right)dy$$

$$H_1=\delta_1{}^*/\theta_1$$

This relationship assumes that several terms involving turbulent velocity fluctuations will be accounted for by the auxiliary equations used and that longitudinal curvature is mild. If the function $U_{p,\delta}(x)$ is known either from experiment or from a potential flow portion solution, two other equations in addition to Eq. (1) are needed to determine the variation of θ, H, and c_f with x. Five different sets (see Ref. 5) of auxiliary equations were selected from the literature and, together with Eq. (1) and the experimentally[5] determined variation of $U_{p,\delta}(x)$, were solved on a digital computer with the aid of the Runge-Kutta-Gill method.[6] This was done in order to evaluate the various sets of auxiliary equations. Based on this evaluation, appropriate auxiliary equations—namely, Eq. (2) Ref. 7, Eq. (3) Ref. 5, and Eq. (4) Ref. 8 for the annulus outer wall and Eq. (5) Ref. 9 and Eq. (6) Ref. 10 for the annulus inner wall—were combined with Eq. (1) and the variation of $U_{p,\delta}(x)$ obtained from a potential flow solu-

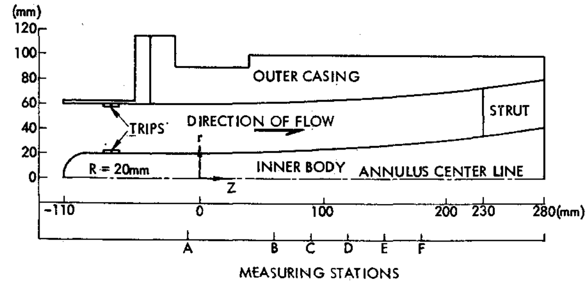

Fig. 1 **The diffusing passage.**

Table 1 Ratio of the radii of longitudinal and transverse curvatures to the boundary-layer thickness

Section (see Fig. 1)	r_l/δ_r Outer wall	r_l/δ_r Inner wall	r_o/δ_r Outer wall	r_o/δ_r Inner wall
A	∞	∞	13.3	4.4
B	445.0	445.0	9.4	3.8
C	313.5	364.8	10.9	3.1
D	352.8	287.3	7.8	3.2
E	246.0	269.0	8.0	3.4
F	246.0	289.2	8.0	4.0

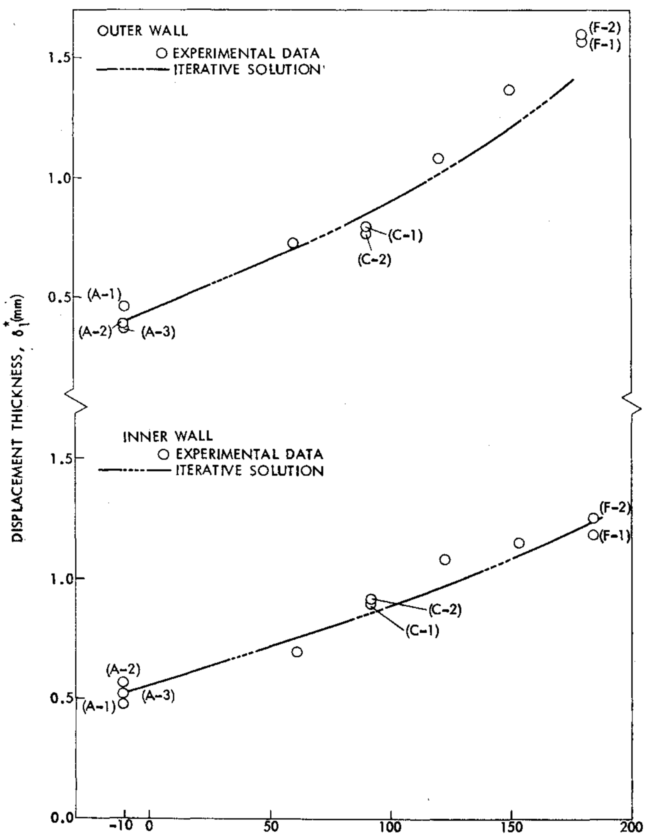

Fig. 2 **Comparison of calculated and measured values of boundary-layer displacement thickness.**

tion based on the streamline curvature method[11] to form an iterative solution. These auxiliary equations are

$$c_f = (0.246)(10^{-0.678H})(R_\theta^{\ -0.268}) \qquad (2)$$

$$\theta \frac{d\bar{H}}{dx} = \bar{H}\left[(H-1)\frac{\theta}{U}\frac{dU}{dx} - \frac{c_f}{2}\right] + 0.0112\, R_\theta^{\ -1/6} \qquad (3)$$

$$\bar{H} = (1.02 + 0.87H + 0.095H^2)/H \qquad (4)$$

$$\theta \frac{dH}{dx} = e^{4.680(H-2.975)}\left[-\frac{\theta}{U}\frac{dU}{dx}\frac{4}{c_f} - 2.035(H-1.286)\right] \qquad (5)$$

$$c_f/2 = 0.0288/[\log_{10}(4.075U\theta/\nu)]^2 \qquad (6)$$

In all instances, values of displacement thickness, shape factor, and $U_{p,\delta}$ at the entrance of the curved passage were obtained experimentally. A sketch of the curved annular diffusing passage is shown in Fig. 1. The results[5] of this analytical solution appear to be fair considering the approximations made. For example, the calculated and measured displacement thicknesses comparison is shown in Fig. 2. Table 1 indicates the significance of the passage longitudinal and transverse curvatures. Bradshaw[12] suggests that longitudinal curvature associated with a value of $r_l/\delta < 300$ is significant. Cebeci[13] states that when the radius of a body in a viscous flow is of the same order of magnitude as the thickness of the boundary layer, the transverse curvature effect on skin friction becomes appreciable. Further improvements in the present calculation method will probably be obtained when the auxiliary equations are revised to more nearly reflect curvature effects. Research along these lines is in progress.

References

[1] Jansen, W., "The Application of End-Wall Boundary-Layer Effects in the Performance Analysis of Axial Compressors," 67-WA/GT-11, 1967, ASME.

[2] Stratford, B. S., "The Use of Boundary Layer Techniques to Calculate the Blockage From the Annulus Boundary Layer in a Compressor," 67-WA/GT-7, 1967, ASME.

[3] Mellor, G. L. and Wood, G. M., "An Axial Compressor End-Wall Boundary Layer Theory," 70-GT-80, 1970, ASME.

[4] Rouse H., *Advanced Mechanics of Fluids*, Wiley, New York, 1958,

[5] Fujii, S. and Okiishi, T. H., "Development of Turbulent Boundary Layers Along the Curved Walls of an Annular Diffusing Passage," Rept. ISU-ERI-AMES-71033, Sept. 1971, Engineering Research Institute, Iowa State Univ., Ames, Iowa.

[6] Moriguchi, S., *Introduction to FORTRAN IV*, Tokyo University Press, Tokyo, Japan, 1961.

[7] Ludwieg, H. and Tillmann, W., "Investigation of the Wall-Shearing Stress in Turbulent Boundary Layers," TM 1285, 1950, NACA.

[8] Moses, H. L. and Chappell, J. R., "Turbulent Boundary Layers in Diffusers Exhibiting Partial Stall," *Transactions of the ASME, Journal of Basic Engineering*, Sept. 1967,

[9] von Doenhoff, A. E. and Tetervin, N., "Determination of General Relations for the Behavior of Turbulent Boundary Layers," Rept. 772, 1943, NACA.

[10] Squire, H. B. and Young, A. D., "The Calculation of Profile Drag of Airfoils," Great Britain Aeronautical Research Council R and M 1838, 1937,

[11] Fujii, S. and Uno, T., "Streamline-Curvature Approach to Duct Flow Problems," TR-140, 1967, Japan National Aerospace Lab.

[12] Bradshaw, P., "The Analogy between Streamline Curvature and Buoyancy in Turbulent Shear Flow," *Journal of Fluid Mechanics*, March 27, 1969.

[13] Cebeci, T., "Laminar and Turbulent Incompressible Boundary Layers on Slender Bodies of Revolution in Axial Flow," *ASME Transactions, Journal of Basic Engineering*, Sept. 1970.